**Some Harder trigonometry problems**

**1.** Let , find the value of .

**2.** Let , and assume . Find .

**3.** Compute .

**4.** **(a)** By using induction on , where x, y ≥ 0 .

 prove the Power Mean inequality: .

 **(b)** Prove that .

**5.** If , find the value of :

**6.** Let

 Find the value of

**7.** Let

 Prove that .

**8.** Let , find the value of

 .

**9.** Compute : .

**10.** Find the sum of the series:

**1.** From given:

2. **Method 1**

 , since , we may neglect the negative root .

 Now,

 **Method 2**

 Let , than

 ∴

**3.** Obvious that are roots of the equation

 .

 , by Binomial Theorem

 Compare imaginary parts,

 are roots of

 are roots of

 Product of roots

 Since

 We have

**4.** **(a)** Let P(n): , where x, y ≥ 0 .

 For P(1),

 For P(2),

 Assume P(k - 1) and P(k) are true for some k∈**N** .

 That is,

 For P(k + 1),

 and P(k + 1) is true.

 By the second principle of Mathematical Induction, P(n) is true ∀n∈**N**.

 For Power Mean inequality, from above

 , by deduction, hence induction.

 **(b)** (i) Since

 Adding, we get

 (ii) By the Power Mean inequality in (a),

**5.** Let

 Then

 ∴

 Similarly,

**6.**

∴

**7.** Let ,

 ∴

**8.** Let , then

 Also, …. (\*)

 , by (\*)

**9.**

 , by Binomial Theorem

 Compare imaginary parts,

 Compare real parts,

 , where .

 are roots of the equation ,

 Or

 are roots of the equation

 , where the root t = 0 is neglected.

 Sum of roots =

 Sum of pair of roots =

 ∴

**10.**

 ∴

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