**Some Harder trigonometry problems**

**1.** Let , find the value of .

**2.** Let , and assume . Find .

**3.** Compute .

**4.** **(a)** By using induction on , where x, y ≥ 0 .

prove the Power Mean inequality: .

**(b)** Prove that .

**5.** If , find the value of :

**6.** Let

Find the value of

**7.** Let

Prove that .

**8.** Let , find the value of

.

**9.** Compute : .

**10.** Find the sum of the series:

**1.** From given:

2. **Method 1**

, since , we may neglect the negative root .

Now,

**Method 2**

Let , than

∴

**3.** Obvious that are roots of the equation

.

, by Binomial Theorem

Compare imaginary parts,

are roots of

are roots of

Product of roots

Since

We have

**4.** **(a)** Let P(n): , where x, y ≥ 0 .

For P(1),

For P(2),

Assume P(k - 1) and P(k) are true for some k∈**N** .

That is,

For P(k + 1),

and P(k + 1) is true.

By the second principle of Mathematical Induction, P(n) is true ∀n∈**N**.

For Power Mean inequality, from above

, by deduction, hence induction.

**(b)** (i) Since

Adding, we get

(ii) By the Power Mean inequality in (a),

**5.** Let

Then

∴

Similarly,

**6.**

∴

**7.** Let ,

∴

**8.** Let , then

Also, …. (\*)

, by (\*)

**9.**

, by Binomial Theorem

Compare imaginary parts,

Compare real parts,

, where .

are roots of the equation ,

Or

are roots of the equation

, where the root t = 0 is neglected.

Sum of roots =

Sum of pair of roots =

∴

**10.**

∴

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